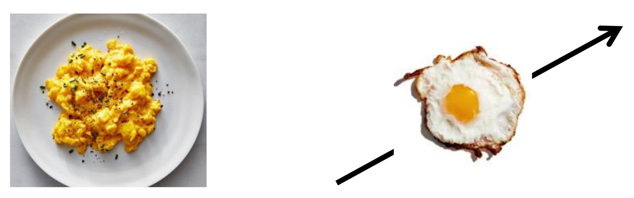
# Spatial Extremal Dependence in Natural Hazard Footprints


Laura C. Dawkins[1] & David B. Stephenson[2]


## 1. Motivation

- Re-insurers use **catastrophe models** to estimate risk of natural hazards for insurance pricing. These models require **synthetic simulations** of natural hazard events for validation
- **We develop a statistical model** to represent the relevant properties of the hazard footprint, allowing for **very quick simulation** of synthetic events
- Hazard losses are sensitive to the **dependency between extreme values of the hazard variable at different spatial locations.** The statistical model must therefore accurately represent this **extremal dependence**

Storm damage in Dawlish, UK 2014



Modelling the scrambled egg (hazard footprint), rather than the fried egg (weather system)!

## 2. Application

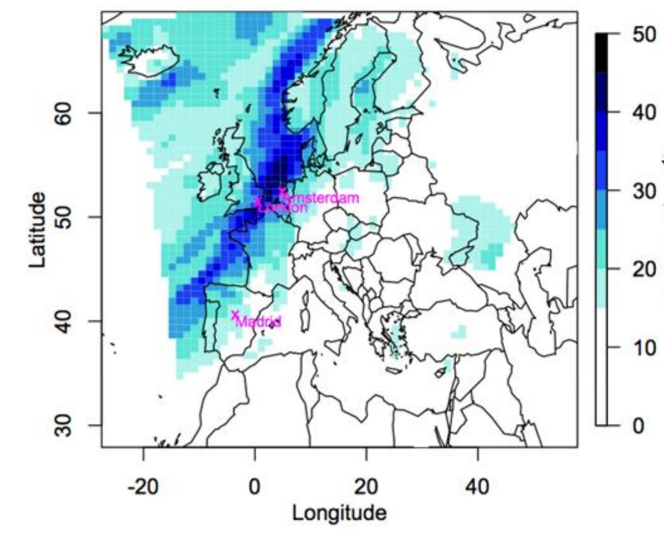
**Windstorm Footprint:**
The maximum 3-second wind-gust speed at each location in the 72 hour period covering the passage of the storm, centred on the time at which the maximum wind speed over land occurs

**Data set:**

- **6103 storms,** from extended winters (October-March) 1979-2014
- Dynamical downscaling of ERA-Interim reanalysis data using the Met Office **25km resolution** North Atlantic-European operational numerical weather prediction model (Roberts et al., 2014)

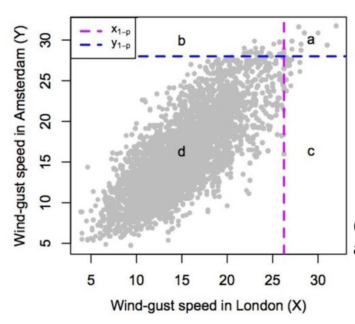
**Footprint for the Great Storm of October '87 (15th – 17th October 1987)**

Caused £4 billion of insured losses in Europe



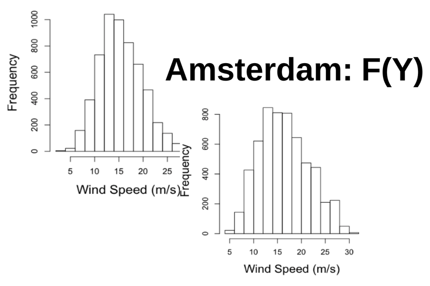

## 3. Exploring Extremal Dependence

The **joint distribution** of footprint wind-gust speeds at two locations, denoted X and Y, can be though of in terms of…



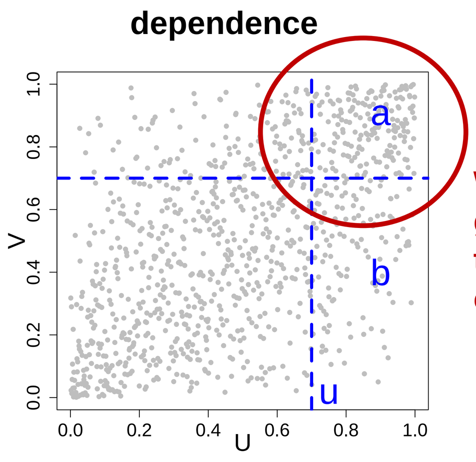

=

…the distributions of gusts at each location …




+

… and their mutual dependence




What is going on in this top corner?

Either **Asymptotic Independence:** Largest values of the variable **rarely occur together**

Or **Asymptotic Dependence:** Large values of the variables tend to **occur simultaneously**

### Method 1: Extremal Dependence Measures (Coles et al. 1999)

$$\chi(u) = \Pr(V > u | U > u) = \Pr(V > u, U > u)/\Pr(U > u) = a/(a+b)$$

$$\bar{\chi}(u) = \frac{\log((a+b)/n)}{\log(a/n)} - 1$$

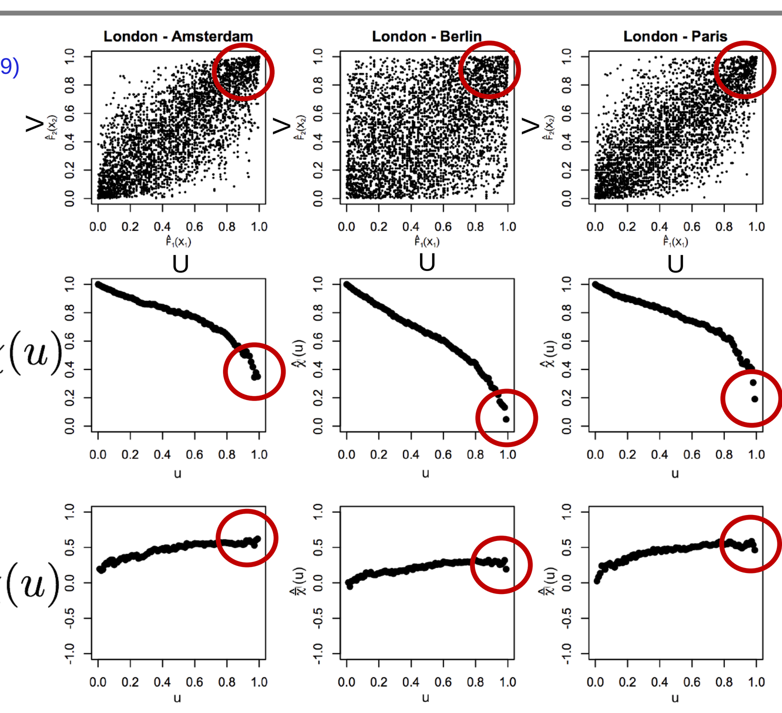
Empirical estimates suggest **asymptotic independence**, but these cannot represent the asymptotic limit u→1

**Asymptotic Independence:**

$$\lim_{u\to 1} \chi(u) = 0 \quad \lim_{u\to 1} \bar{\chi}(u) < 1$$

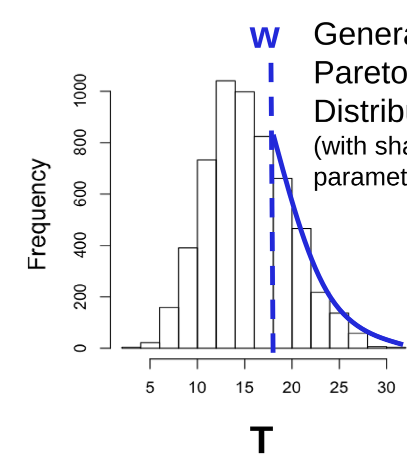
**Asymptotic Dependence:**

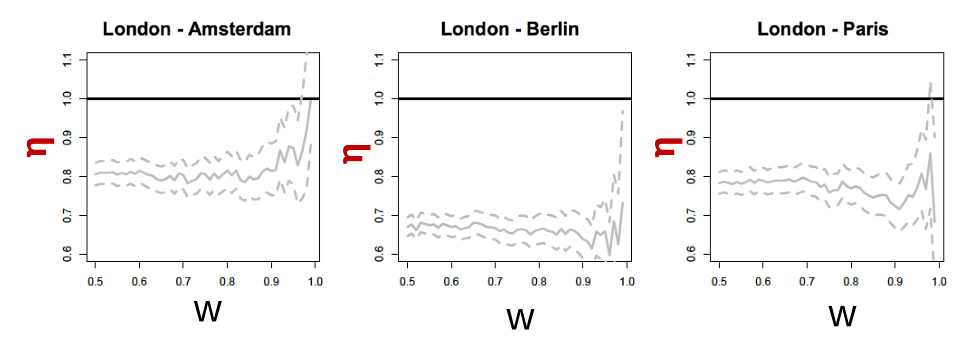
$$\lim_{u\to 1} \chi(u) > 0 \quad \lim_{u\to 1} \bar{\chi}(u) = 1$$




### Method 2: Coefficient of Tail Dependence (Ledford and Tawn, 1996)

1. Transform wind gust speeds at each location to follow a Laplace distribution;
2. For each pair of observations take the minimum, denoted **T**;
3. Fit a univariate extreme value Generalised Pareto Distribution to **T**, above a high threshold **w**;
4. This distribution has three parameters: location, scale and shape;
5. **The shape parameter, η, of this distribution characterises extremal dependence, representative of the limit** u→1;
6. Repeat for a range of values of **w** to explore sensitivity to this choice of threshold



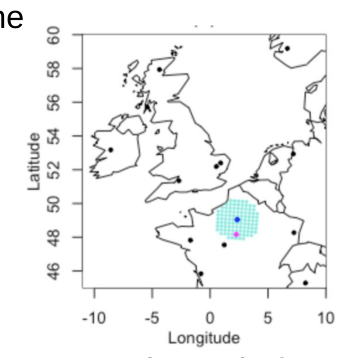




**η <1** Asymptotic independence

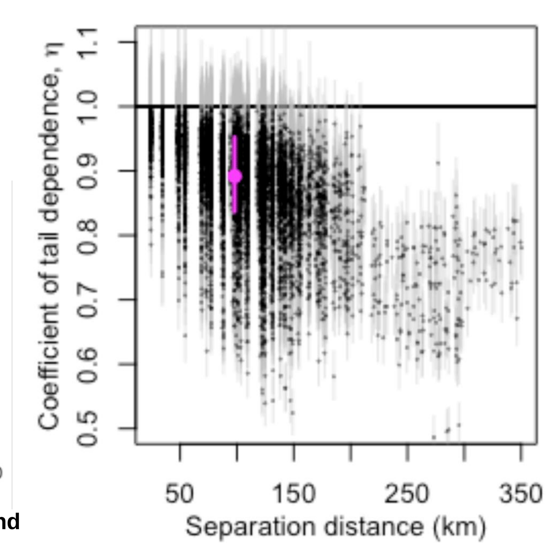
**η =1** Asymptotic dependence

For these 3 pairs of locations **η**<1 for most w's in the range 0.5-1, suggesting **asymptotic independence**. What about other pairs of locations?

- Take a stratified sample of 100 locations in the European footprint domain
- Estimate **η** for each of these locations, paired with the 100 nearest locations
- Explore **η** for increasing separation distance




Example central point and 100 nearest locations




- Indication of **extremal dependence** for some very nearby locations
- **Asymptotic independence is the dominant extremal dependence structure** across the spatial domain (200 – 3500km all **η**<1)

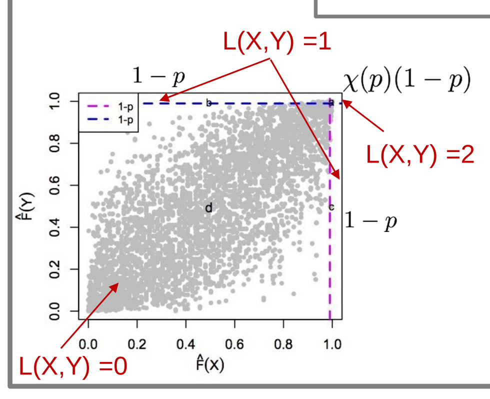
## 4. Natural Hazard Loss Diagnostic

### Bivariate windstorm loss model (Roberts et al. 2014)

- Loss occurs when gusts exceed a **location specific loss threshold**, the **p=0.99** quantile footprint wind gust speed (1979-2014)

$$\boxed{L(X,Y) = 1(X > u(X)) + 1(Y > u(Y))}$$

where 1(x) is an indicator function



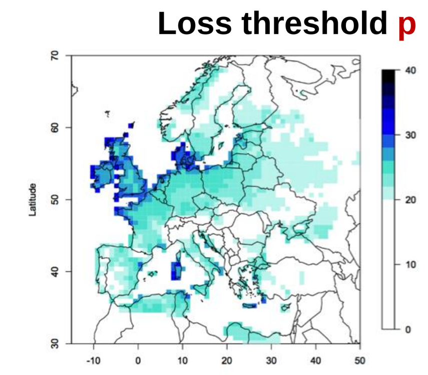

$$\chi(p) = Pr(\hat{F}(Y) > p | \hat{F}(X) > p)$$

$$\Pr(L(X,Y) = 2) = \chi(p)(1-p)$$

$$\Pr(L(X,Y) = 1) = 2(1-\chi(p))(1-p)$$

$$\Pr(L(X,Y) = 0) = (\chi(p) - 1)(1-2p)$$

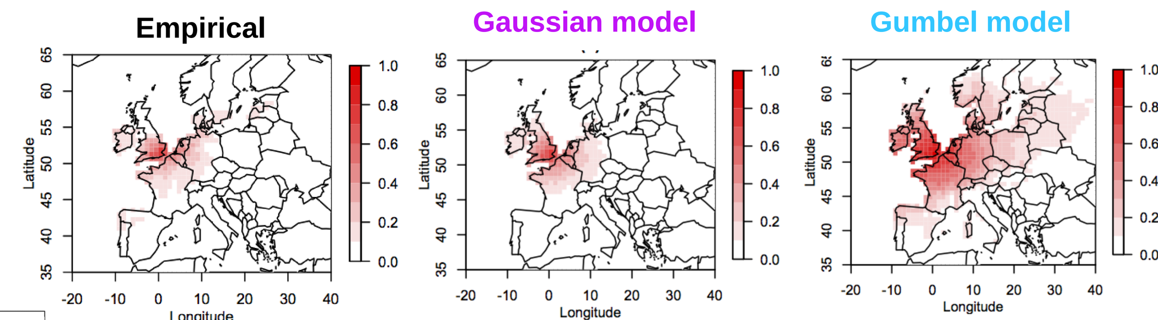
**Loss threshold p**



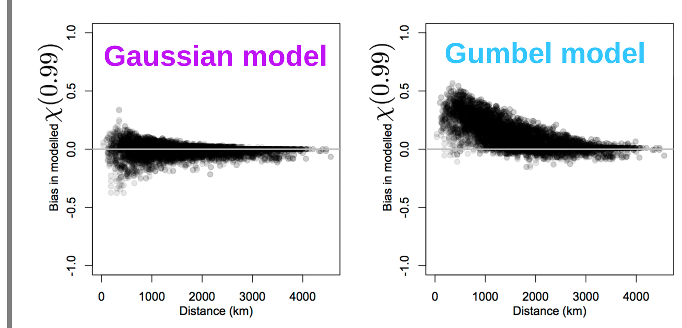
The windstorm **loss distribution** for a pair of locations **can be expressed in terms of χ(p)**, where p=0.99 quantile wind gust at each location

### Extremal dependence and Loss

- The **Gaussian** dependence model characterises asymptotic independence
- The **Gumbel** dependence model characterises asymptotic dependence




Maps of χ(0.99) for London paired with all other locations in Europe




Bias: χ(0.99, Gaussian/Gumbel) - χ(0.99, Empirical) for all combinations of pairs of 100 sampled locations in Europe

- **The Gaussian model (asymptotic independence) represents the empirical joint losses well**
- The Gumbel model (asymptotic dependence) over-estimates dependence between pairs of locations, particularly those <1000km apart, resulting in over-estimation of joint losses
- This gives further evidence of **asymptotic independence** in windstorm footprint wind gusts

## 5. Why are windstorm footprints asymptotically independent?

- **Turbulent wind fields** can be efficiently and realistically simulated using **stochastic processes**
- This approach is widely used for many applications such as testing loads on new aircraft designs

→

- Such models assume that the Cartesian velocity components at location j:

$u_j$ and $v_j$ such that $X_j = \sqrt{u_j^2 + v_j^2}$,

are **independent Gaussian processes**, each with a prescribed spatial covariance function

→

- Since the individual velocity components are bivariate normal, they are **asymptotically independent** at different locations
- e.g. $u_1$ and $u_2$ are asymptotically independent when locations j = 1 and j = 2 differ, and likewise for $v_1$ and $v_2$

→

- Furthermore, it can be shown that the square of each velocity component is also **asymptotically independent**
- See Appendix of Dawkins and Stephenson (2018)

→

- The squared wind speeds at pairs of locations are sums of two such independent components:

$$(X_1^2, X_2^2) = (u_1^2 + v_1^2,\ u_2^2 + v_2^2)$$

- **Asymptotic independence** suggested by simulation in Dawkins and Stephenson (2018), now proven.

## 6. Conclusion

- **Asymptotic independence** is the dominant form of extremal dependence in the 6103 dynamically-downscaled European windstorm footprints
- This is supported by the properties of stochastic models for turbulence
- The **Gaussian** model appears to fit the data well, which allows for fast geostatistical simulation of footprints
- For further details see Dawkins and Stephenson (2018)


**References**

**Dawkins L. C. and Stephenson D. B. (2018). Quantification of extremal dependence in spatial natural hazard footprints: independence of windstorm gust speeds and its impact on aggregate losses. Natural Hazards and Earth System Science, 18, 2933-2949**

Coles, S., Heffernan, J., and Tawn, J. (1999). Dependence Measures for Extreme Value Analysis, Extremes, 2, 339–365.

Ledford, A. W. and Tawn, J, A. (1996). Statistics for near independence in multivariate extreme values. Biometrika, 83:169-187.

Roberts, J. F., Champion, A., Dawkins, L., Hodges, K. I., Shaffrey, L., Stephenson, D. B., Stringer, M., Thornton, H. and Youngman, B., (2014). The XWS open ss catalogue of extreme windstorms in Europe from 1979–2012. Natural Hazards and Earth System Science, 14:2487–2501.



[1] **Met Office** FitzRoy Road, Exeter, Devon, EX1 3PB United Kingdom
Tel: +44 7525388589
Email: laura.dawkins@metoffice.gov.uk

[2] **University of Exeter** College of Engineering, Mathematics and Physical Sciences, Exeter, United Kingdom